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I. Main duties of the research unit in 2014

The fundamental goal of the Alfréd Rényi Institute of Mathematics is to pursue research of high international standing in pure mathematics. The institute is an important center of mathematics internationally. In 2014 it further improved its reputation both in Hungary and abroad, thanks to the outstanding achievements of its researchers. At the International Congress of Mathematicians 2014 three researchers working in the institute (at least partly) gave invited lectures. One researcher was elected to Academia Europaea and others have received several international prizes in 2014: Cole Prize (American Mathematical Society), Synge Prize (Royal Society of Canada), Doctor Honoris Causa (University of Buenos Aires). One researcher has obtained a five-year support of the European Research Council in the Consolidator Grant category. This will be the sixth research group in the institute supported by an ERC grant. Quite a number of national awards were also received by members of the institute (including emeriti): two of them obtained the Széchenyi Prize, other researchers got the Academy Prize, the Prima Prize, the Paul Erdős Prize and the Béla Gyires Prize of the Section of Mathematics of the Hungarian Academy of Sciences, the Young Researcher Award of the Academy, as well as the "Bonis Bona – for the talents of the nation" prize. One scientist obtained the Doctor and Professor Honoris Causa title from the Eötvös University. Young researchers were distinguished by the Géza Grünwald medal and the Gyula Farkas prize of the János Bolyai Mathematical Society.

The scientific tasks of the institute concentrate on fundamental research. However, significant efforts were devoted to some topics of applied mathematics as well. The main applied areas investigated in the institute are cryptography, the theory of large networks, as well as bioinformatics, but mathematical statistics has also been applied in several related areas (medical research, environmental studies, etc.). The institute is organized in the framework of 9 scientific departments and 3 Lendület research groups. The Cryptography group started in 2009 works in the traditional organizational form inside a scientific department, the three newer ones (Low Dimensional Topology since 2010, Groups and Graphs since 2012, Large Networks since 2013) work organizationally independently, each enjoying the support of ERC as well. The research topics of the institute are continuously adjusted to the most recent developments in mathematics. In 2014 a small group on mathematics education was formed in the institute, and later this group obtained the support of the Academy. The main task of the group is continuing the programs on fostering mathematical talents; in addition they are planning programs for underprivileged students as well.

II. Outstanding research and other results in 2014

a) Outstanding research and other results

Low Dimensional Topology Momentum Research Group

A book on grid homology is nearing its completion. Besides the fundamentals of the theory it contains new proofs of classical and modern results.

By perturbing the previous definition, a 1-parameter family of knot homologies has been defined, and this provided a 1-parameter family of numerical knot invariants. These invariants then can be used to show that the subgroup of the concordance group consisting of topologically slice knots contains an infinite rank free Abelian direct summand.

In high dimensional contact topology it has been shown that for the Stein cobordism as ordering there is always a maximal almost contact element. In dimensions $(8k - 1)$ the spheres (for k at least 2) admit almost contact structures which contain no Stein fillable contact structures.

A generalization of J-holomorphic disk techniques from dimension 3 to higher dimension gave results about subcritical contact surgeries. Similar ideas led to the discovery that the isomorphism group of a contact manifold can be disconnected inside the connected component of the diffeomorphism group.

In a different direction, uniqueness of the quasi-polynomial of counting function associated with Hilbert series of negative definite plumbing trees was shown. In addition, a surgery type formula for quasi-polynomials were found.

Closed 4-manifold can be encoded by certain configurations of curves on a closed, orientable surface; for low genera these configurations and the corresponding 4-manifolds have been classified.

They examined some questions regarding the Alexander polynomial of singularities on rational cuspidal complex projective plane curves. They proved a conjecture from 2006 in the special case of curves with at most two cusps and by finding a couneterexample, they disproved the conjecture for tricuspidal curves.

They gave an explicit condition for embedded circles in two-sphere being the multiple point set of an immersion into three-space. They showed that 4-manifolds with simple singular circle fibrations admit positive scalar curvature metric.

Groups and Graphs Momentum Research Group

They studied the expected spectral measure of the 1-dimensional Schroedinger operator and for low noise, the behavior of a typical eigenvector. They gave a new bound on the Hölder continuity of the expected spectral measure for low noise.

The matching measure of an infinite vertex-transitive graph can be defined in two possible ways: either as the spectral measure of the tree of self-avoiding walks of the graph, or as the weak limit of the root measures of matching polynomials for a sequence of graphs converging to the graph in the Benjamini-Schramm sense. They showed that these definitions are equivalent, which allows to more effectively bound the monomer-dimer free energy of Euclidean lattices than what would be possible using the classical theory of Mayer series.

They showed that the set of factor of IID processes on the d-regular tree is not closed in the weak topology. The result generalizes to any transitive graph with uncountable spectrum. Their proof uses a new construction of Gaussian wave eigenfunctions.

The Positive Graphs Conjecture states that whenever a graph is positive (i.e., its homomorphism number into any weighted target graph is nonnegative) then it is also symmetric (i.e., it can be produced by gluing together two copies of the same graph along an independent set of vertices). With collaborators they showed that if all positive quantum multigraphs can be written as a countably infinite sum of squares of labeled quantum multigraphs then the Positive Graphs Conjecture is true.

They started to investigate a new interpretation of sofic entropy theory. Their ongoing work already gives a strong simplification of the known results in the literature, in particular, the breakthrough theorem of Bowen, who proved the Kolmogorov-Sinai theorem in the realm of sofic groups.

Large Networks Momentum Research Group

The understanding of large networks and structures, the description and the forecasting of their behaviour plays an important role in various areas of science nowadays (e.g., in the research of interactions of proteins and genes, the human brain, social networks, the internet). The work of the “Large Networks” Momentum Research Group is related to these questions; the main goal is to find continuous limiting structures for large systems (which may contain only partial information) such that the theoretical, analytical results on this limit may be translated to the original finite system, in order to better understand its structural properties.

One of the best models of real world networks may be graphs. In this case it depends on a (maybe random) process whether two nodes are connected to each other. The main area of research in 2014 was that of the extension of the theory of dense graph limits, with the aim to give it an extended, functional analytic foundation. The ability to pass from the concrete graph theoretic applications to the abstract functional analytic framework and back again ended up playing a key role. The main motivation was provided by a class of random graph models that arise in several applications directed towards real world networks, but not covered by the previously known theoretic results. The main result was finding a framework and abstract formulation that made it possible to also cover the above mentioned class of random graph models, yielding a unified context in which to treat the until now sporadic outlying special cases.

Randomized local algorithms may be important both in computer science and the examination of systems consisting of many animals or organisms. First, the nodes of the network randomize certain quantities independently of each other, then every node makes a decision, based on the information in a given neighbourhood. The question is whether the (possibly) billions of nodes of the network may cooperate globally if (according to the spatial structure) each element can communicate only with the hundred closest others. Regarding this, the possible correlation structures of the randomized algorithms (factor of i.i.d. processes) were described on graphs which do not contain any cycles, and whose nodes all have the same number of connections. Furthermore, a necessary condition was given for processes that can be local (or approximated with local processes) in the above sense.

Fourier analysis is similar to the process when human brain decomposes noise into sounds of different frequency. A similar decomposition is one of the goals of higher order Fourier analysis in a much more general and abstract setup. The theoretical foundation of this area was continued in the research group. As applications to this, an algorithm was invented, which predicts some kind of time series. Software was developed to analyze the quadratic Fourier structures in time series. Moreover, these softwares measure the changes of the strength of quadratic Fourier structure in different time intervals. This may be used for predicting financial time series for example.

Didactics Research Group

The Didactics Group was established officially in July of 2014.

They organized 7 weekend maths camps for talented students in Fall 2014. Students spend about 14 hours with mathematics during weekend in these camps. Cca. 200 students visited at least one of these weekend camps.

In the Summer they organized two summer maths camps: MaMuT (Camp of Mathematical Amusements) and MaMuT2. Talented students from age 10 to 18 with outstanding results in Hungarian and international mathematical competitions were invited to improve their mathematical knowledge and skills.

3 events were held by them in high schools in order to popularize mathematics and discovery learning of mathematics.

Their project proposal “The Education of the Gifted Using Methods for Inspiration to Discover Things in Teaching Mathematics” was awarded by Hungarian Academy of Sciences Presidential Committee for Public Education.

Department of Algebra

Toric ideals of smooth flow polytopes were studied. In the special case when all of the vertices are 0-1 vectors it was proven that, if a sufficiently large proportion of the vertices are smooth then the corresponding toric ideal is generated in degree two. Moreover, if all but at most one of the vertices are smooth then the toric ideal possesses a quadratic Gröbner basis.

A new definition for prime congruences in additively idempotent semirings was given using so-called twisted products. It was shown that this class exhibits analogous properties to the prime ideals of commutative rings. An explicit description of prime congruences was given for the polynomial semirings over the tropical semifield and the two element idempotent semifield. The Krull-dimensions were calculated in all of the studied cases. Applying these tools an improvement of a 2013 result was proven, which can be regarded as a tropical analogue of the Nullstellensatz.

Conjectures of Schwarz and Hunziker about the subalgebra of vector-invariants generated by polarizations were proved in special cases.

The maximal possible dimension of a commutative subalgebra in the Grassmann algebra (exterior tensor algebra) of a finite dimensional vector space was determined. A linear algebra process assigns to a commutative subalgebra an equidimensional commutative subalgebra spanned by monomials. This allows to reduce the problem to a question about extremal intersecting set families. Partial information was obtained on the structure of maximal dimensional commutative subalgebras, and examples of maximal (with respect to inclusion) but not maximal dimensional subalgebras were given.

Moduli spaces of quiver representations were investigated in the toric case. It was pointed out that there are only finitely many such toric varieties in each dimension. A scheme for their classification was developed. This raises the question whether the above finiteness result remains valid in the general (not necessarily toric) case. It was shown that the answer to the affine analogues of this question is affirmative.

It is known that the ideal class group of the ring of polynomial invariants of a finite group is isomorphic to the subgroup of the character group consisting of the characters that vanish on all pseudoreflections. A new proof of this result was obtained by explicitly identifying the natural invariant theoretic interpretation of the divisor theory predicted by the general theory of Krull monoids.

The submodule lattices of rank at least three free modules were investigated. Projectivities and dualities, i.e., both lattice isomorphisms and anti-isomorphisms were discussed. Rings such that the corresponding submodule lattice is selfdual were characterized.

It has been shown that the Huq and the Smith commutator do not coincide in the variety of near-rings – the connection of these commutators is an open question in categorical commutator theory. Further results have been obtained on Morita equivalence of semigroups without local units (in the framework of Estonian–Hungarian Academy exchange program).

Department of Algebraic Geometry and Differential Topology

They studied Riemannian manifolds in which the volume of a tube of a given radius around a curve depends only on the length of the curve and of the radius whenever the radius is small. They proved that all the harmonic spaces have this property, and computed explicit formulae for the volume of tubes in Damek-Ricci spaces.

They studied the algebraic structure of finite groups acting on a manifold. They found a counterexample to a conjecture of E. Ghys, proposing that any finite group acting on a compact manifold has an Abelian subgroup of bounded index, where the bound depends only on the manifold. They proved a weaker version of Ghys' conjecture about the existence of a soluble subgroup of bounded index.

They analysed major properties of the lattice cohomology of plumbed 3-manifolds. The main result establishes a theorem which reduces the rank of the lattice in the definition of the cohomology theory to the number of “bad vertices” of the plumbing graph. Several application are listed, mostly related with the topological series (zeta functions) associated with the graph: they proved that the multivariable periodic constant of the reduced series still can be identified with the Seiberg-Witten invariant.

They proved that for several hypersurface singularities the geometric genus is the Euler characteristic of the path-0-lattice cohomology. The main cases include germs with Newton nondegenerate principal parts, and superisolated singularities.

They introduced several spaces associated with holomorphic arcs of a local analytic germs. They determined the connected components of the space of short holomorphic arcs. This is a generalization of the famous Nash conjecture.

For holomorphic germs defined on normal surface singularities, they proved semicontinuity properties of the Hodge spectrum using topological methods. The main tool is the newly introduced Seifert matrix (adapted to this case).

They related the topological notion of the Smale invariant associated with immersions of the 3-sphere into the 5-sphere with holomorphic invariants of maps from the complex 2-space to 3-space. They characterized the Smale invariant via an analytic invariant. In this way they answered an old question of Mumford.

They proved that Nahm transform is a hyper-Kaehler isometry between moduli spaces of parabolic Higgs bundles on the Riemann sphere.

They proved the existence of a birational isomorphism between moduli spaces of logarithmic connections and Hilbert schemes of points on certain ruled surfaces. This is related with the famous Deligne-Simpson problem.

According to a theorem of Ribet an Abelian variety defined over the maximal cyclotomic extension of a number field has only finitely many points of finite order. They have extended this theorem to elements of finite order in odd degree étale cohomology groups using more recent advances in p-adic Hodge theory.

Reached sharp results regarding representability of cohomology classes by immersions. Calculated numerous incidence coefficients for Morin singularities and identified them with differentials of a spectral sequence that abuts to the stable homotopy groups of complex projective spaces. Investigated this latter identification both geometrically and algebraically, and proved structural results that make related computations easier.

Department of Algebraic Logic

By formalization of a symmetry axiom (which follows form the Galilean principle of relativity) and some natural assumptions of kinematics, they gave a first-order logic axiom system, and they characterized the groups of world-view transformations that occur in models of this axiom system. These groups are special subgroups of the Poincaré group or the Galilean group or the isometry group of the Euclidean space. It is important to note that this space is based on an arbitrary ordered field in place of the real numbers, because in the former case the Poincaré, Galilean and the isometry groups have a much diverse variety of subgroups.

They have shown that the axioms of special relativity can be interpreted into Newtonian kinematics, via using light signals to redefine the coordinate systems of Newtonian observers. They have also shown that if we remove the non-slower-than-light observes from Newtonian kinematics and add “Ether” as a basic concept to special relativity, then the interpretation can be “inverted”, i.e., the two theories become definitionally equivalent.

They wrote up a complete and decidable axiom system in first-order temporal logic that can express the basic paradigmatic relativistic effects of kinematics such as time dilation, length contraction, twin paradox, etc. In this axiom system, the coordinatization is definable using metric tense operators with signaling procedures. These operators correspond to inertial agents conducting signaling experiments, while drifting in space, in order to explore the spacetime they live in.

They constructed, for each natural number n greater than 2, a “circular” first-order logic definition using only n variables from which the “circularity” cannot be eliminated if we cannot use more than n variables, in spite that in each model of the background theory the definition singles out exactly one relation. The existence of such a definition was not known for $n = 4$ and the given construction unifies and simplifies the previously known ones.

They proved that standard dynamic logic is *ZF*-absolute but not *KPU*-absolute. This non-absoluteness means that properties of the set-theoretical tools for formalizing the logic are entangled with properties of the phenomenon, dynamic logic, we want to model. Dynamic logic is a logic for reasoning about consequences of actions (e.g., verification of programs, or outcomes of physical experiments).

Department of Analysis

The famous difficult conjecture of Ryser concerning the cyclic Hadamard matrices has been successfully reformulated in terms of Walsh-Fourier analysis. This established a new approach to solving the problem.

By modifying the Delsarte method a better upper bound was given for the density of measurable plane sets whose points are not at distance 1 from each other. The previously known upper bound was improved, and with some natural additional assumptions the Erdős conjecture stating that the density of such sets can not be larger than ¼ was verified. In addition, it was shown how the Brunn-Minkowski inequality leads to estimates in the higher dimensions.

It has been investigated on which sets the standard 1-dimensional Brownian motion is monotone, of bounded variation, or alpha-Hölder continuous. The Hausdorff dimension of their level sets equals ½, it has been verified that larger dimension can not be attained.

The conjecture that if a system of lines is built by extending a system of segments then this does not change the Hausdorff dimension has been investigated. It was shown that this conjecture yields the famous Kakeya conjecture on the box dimension. In addition the conjecture was verified on the plane.

With respect to the Christoffel functions which play a fundamental role in the theory of orthogonal polynomials it was shown how their size around the boundary of the multidimensional star-like and convex domains depends on the smoothness of the boundary of the domains.

The Bernstein type convergent interpolatory process has been extended for the case of Jacobi nodes.

The existence, uniqueness and asymptotic properties of the positive solutions of nonlinear difference equations have been given.

Two sided estimates for the Lebesgue function of the Lagrange interpolation based on nodes from disjoint intervals were given. It was shown that there exist nodes from disjoint intervals for which the Lebesgue function of the Lagrange interpolation has at most logarithmic growth.

Department of Discrete Mathematics

They proved the list version of the graph packing theorem of Bollobás and Eldridge. Apart from an additive constant, they proved a conjecture of Zak about graph packings.

They proved theorems about the independence number and strong degree of hypergraphs with no linear cycles.

They determined the structure of the directed graphs such that every edge is contained in exactly three cycles.

They determined the exact Turán number for a certain wide class of graphs and hypergraphs. These investigations might lead to the proof of the Erdős-Sós-Kalai conjecture.

They asymptotically determined the maximum number of copies of a given (small) partially ordered set that can be embedded into the Boolean lattice so that no elements of distinct copies are comparable.

The maximum number of subsets of an n-element set such that every pair of these subsets is non-disjoint has been determined long ago. Now the modified problem was considered: the pairwise unions (of distinct) subsets have to be intersecting. The maximum number of subsets under this condition was determined. One can generalize the problem taking t-wise unions, and replacing the intersecting property with the condition that the intersections (of these t-wise unions) have an intersection of size s. Many cases of this general setting have been also solved.

They improved the general asymptotic upper bound for the forbidden poset problem. They also solved several special cases for posets which were all generalizations of the butterfly poset. Furthermore, they proved a version of the Erdős-Szekeres theorem for partially ordered sets.

They improved the upper bound on the online choosability of complete balanced bipartite graphs.

They studied the properties of local chromatic number of graphs and gave an efficient algorithm for the list coloring of permutation graphs.

They sharply estimated the randomized decision complexity or the recursive majority function.

They proved a fundamental theorem about the design of pseudorandom graphs. It was shown that essential expansion can be forced by local conditions, i.e., a large family of essentially reliable networks is easy to construct. They applied this theorem to give an alternative proof to a topological problem of Freedman.

They investigated some new properties of quasi-random graphs (in some cases depending on the relevant parameters). The proofs are based on the theory of graph limits, and on the method and results developed. These give the possibility to translate a combinatorial problem to an analytic problem, which then can be translated to an algebraic problem.

They introduced the convergence and limit theory of finite dimensional representations. It is an extension of the sparse graph limit theory. Using this method they characterized the amenability of skew fields and proved the soficity of the free skew field. They described the regular closure of the lamplighter group.

They studied questions concerning t-fold $(n-k)$-blocking sets of the n-dimensional projective space of order q. The so called General Linearity Conjecture states that if t is not too big and the size of a t-fold $(n-k)$-blocking set is under a certain bound, then the set is the union of t_i-fold $(n-k)$-blocking sets, with t being the sum of the numbers t_i. They proved the General Linearity Conjecture for a family of t-fold 1-blocking sets.

They continued research on the applications of the Regularity Lemma. They implemented the algorithmic Regularity Lemma. Based on this they developed a new clustering algorithm which they successfully tested on several concrete databases.

They introduced the graph parameter called Dilworth rate that generalizes the information theoretic graph parameter called Witsenhausen rate to directed graphs. Apart from clarifying the close connection with Sperner capacity it has been showed that the dichromatic number introduced by Neumann-Lara in 1982, as well as its fractional version is an upper bound on the Dilworth rate and with the help of this the value of the latter was determined for some specific graphs.

An old anti-Ramsey type problem has a natural analogue concerning graph orientations in place of edge colorings. The question is the minimum number of distinct orientations of the edges of the complete graph on n vertices needed to make every triangle cyclically oriented in at least one of these orientations. The problem has been solved for k-cycles in general for an arbitrary but fixed $k > 2$.

Studying degree sequence problems, they finished their investigations in connection of the existence of the solutions of Joint Degree Matrices. They also proved some partial results on sampling procedures of JDM. They showed that the usual Markov Chain Monte Carlo process is fast mixing on so-called “balanced realizations” of a JDM.

They generalized well-known, classical multi-part extremal set system results for partially ordered sets.

They studied edge decompositions of graphs from several points of view. In the classical area of so-called resolvable decompositions they investigated types in which each parallel class consists of identical components, namely paths or cycles on three vertices, or stars of three edges. For the first two types, on the complete graph or the complete graph minus a perfect matching they determined exactly which combinations of those two class types are possible. Concerning general graphs decomposable into induced subgraphs isomorphic to a given graph, they gave asymptotic bounds on the largest possible number of edges as a function of the number of vertices.

They showed that “most mirrors are even more magic”, meaning that for most convex mirrors in the plane most points see continuum many copies of themselves. Here both “most” is meant in Baire category sense.

They have determined how many affine diameters of a convex body K go through a point of K on average. The number in question turns out an interesting and not continuous parameter of K.

They improved the widely investigated bounds on the box cover number of d-dimensional point sets, their new bound is very close to best possible.

They proved that a finite group of bounded Noether index has a cyclic subgroup of bounded index. For a finite simple group of Lie type the Noether number is at most a power of the group order with exponent less than 1.

Department of Geometry

They proved results about the packing density of some 3D bodies. They found several new families of 3D polyhedra which cannot be triangulated without adding new vertices.

They proved that a complex polynomial p of degree n has all roots inside the unit circle, then the absolute value of $p'(z)$ is greater than $n/2$ times the absolute value of $p(z)$, for every point z on the unit circle.

They showed that the packing density of the n-dimensional regular cross-polytope approaches 0 exponentially.

They proved an asymptotic formula for the expectation of the difference of the volumes of circumscribed random polytopes around a convex body and the convex body and also proved an asymptotic upper bound on the variance of the volume of the circumscribed random polytope, provided that the convex body slides freely in a ball.

They proved an asymptotic upper bound on the variance of the number of vertices of random disc-polygons in convex discs which have C^2 boundary and are intersections of unit circular discs.

They proved that the infimum of the volumes of convex polytopes with given facet areas is 0.

A graph is called semi-algebraic with parameter k if its vertices can be represented by points in R^k such that two vertices are joined by an edge if and only if the coordinates of the corresponding points satisfy at most k polynomial inequalities and equalities of degree at most k. These graphs have more favorable combinatorial properties than most other graphs. They established Turán-type and Ramsey-type results for semialgebraic graphs and hypergraphs with fixed parameter k, which are much stronger than for general graphs. They found a very strong, almost "perfect" version of Szemerédi's Regularity Lemma for such graphs and hypergraphs and applied it to solve a number of geometric problems.

They proved that every arrangement of n lines can be colored with $O\sqrt{n/log\ n}$ colors such that none of the cells have all bounding lines of the same color.

Department of Set Theory and General Topology

They considered decompositions of the real line into pairwise disjoint Borel pieces so that each piece is closed under addition, and they investigated the possible number of pieces. They found a model in which the answer is strictly between countable and continuum.

Given a property P of subspaces of topological spaces, P-bounded spaces are defined as those in which any subspace of property P has compact closure. They studied P-bounded spaces for the following three choices of property P: 1. countable discrete, 2. countable nowhere-dense, 3. second countable. This led to interesting new classes of spaces between countably compact and omega-bounded spaces. They gave examples in ZFC that separate all these boundedness properties and their appropriate combinations.

They defined anti-Urysohn spaces as those Hausdorff spaces in which any to non-empty regular closed sets intersect. They proved that for every infinite cardinal there is an anti-Urysohn space of that size. A topological space is strongly anti-Urysohn if it does not contain two disjoint infinite closed sets. They constructed consistent examples of strongly anti-Urysohn spaces, but the existence of a ZFC example remained open.

They succeeded to give a complete answer to a question well-investigated since the 70's: What are the possible order types of chains of Baire class 1 functions?

Kechris and Louveau developed the theory of ranks defined on the Baire class 1 functions. They extended this theory to the Baire class alpha case, and as an application they solved a problem concerning paradoxical geometric decompositions.

Department of Number Theory

Two members of the Department took part in the Polymath 8 project, which yielded an improvement of Zhang's theorem, which is a weaker form of the more than two thousand year old twin-prime conjecture. The original twin-prime conjecture asserts that there are pairs of prime with difference two beyond every limit. The project succeeded to lower the bound to 246, using both theoretical improvement and extensive computations. By a further development of the above mentioned methods they succeeded to solve some other old number theoretical problems, which were mostly formulated as conjectures 60-70 years ago by Paul Erdős.

Gauss investigated already in the 18^{th} century class number of quadratic fields and the problem when is the class number equal to one. They investigated the classical class number one problem for a special family of real quadratic fields (for the so-called Richaud-Degert fields). They extended the proof given for the Yokoi case to a large subset of the Richaud-Degert fields. This extension of the proof required new ideas as well as heavy computer work.

Additive Combinatorics is one of the most quickly developing field of modern mathematics. It investigates among others problems about sets (often consisting of integers) which arise as the sum, difference or product of two given sets. Members of the department reached results in these directions, for example new results in a classical sum-product type problem. The proofs are partly of combinatorial nature, while others use analytic methods, large sieve type and character sum type estimates to study the multiplicative properties of sum sets. They also considered the distribution of the values of expander polynomials in prime fields. These results arose from the field of theoretical Computer Science and have connection with it. They also investigated problems related to difference sets, with emphasis on variants of Delsarte's method using positive Fourier series. The paper describing the basic ideas appeared already. An other paper on sets on the plane and in higher dimensions avoiding unit distance is in preparation. They developed a quantitative version of the Combinatorial Nullstellensatz.

A very important problem of analytic number theory is the estimation of complex functions. A classical method is to use convexity theorem to derive upper bounds for the order of a complex function in a strip if we know the growth of order on the two bordering vertical lines of the strip. The estimates which go beyond this are called subconvex results and they have a very important role in various branches of number theory. Members of the department succeeded to reach very important and general such results for automorphic functions.

Department of Probability Theory and Statistics

They investigated the Lorenz flow which plays an important role in mathematical physics and meteorology. The main object of this investigation was the rate of mixing of this flow. By an old conjecture this rate of mixing is an exponentially decreasing function of the time. Important progress was achieved in the proof of this conjecture. This research was made jointly with the members of the Seminar group of Ergodic Theory and Dynamical System at the Vienna University and of the EDTS Seminar group of the University of Warwick. Related to this research they proved the exponential decay of the solution of the heat conduction equation related to this model. Another problem where they got new results was about the superpolynomial decay of correlation in the system of two falling balls.

Some hard problems about random walks were also investigated. A limit theorem, an invariance principle and a limit theorem with probability 1 was proved for the so-called random walk with spider structure, which is a random walk model introduced by Walsh in 1978. A useful condition was given which describes when an anisotropic random walk with transition probabilities depending on the sites is recurrent or transient.

Earlier estimates on the asynchronous multiple access channel error exponent have been improved. The study of information-theoretically motivated "multiple priors" risk models in Financial Mathematics has been continued, and new conditions were given for the existence of a distribution that realizes the maximum risk.

Good estimates were given for the mixing time of some special but in some applications important Markov chains. Such models were investigated where a circle was extended with some randomly chosen edges. These results made possible to improve the upper and lower bounds for the time needed to find the optimal strategy in some machine learning models.

They considered some interesting families of random sums of independent random variables, and gave a sharp estimate for their supremum. They could study such models whose investigation is not possible by means of standard methods which are based in some hidden way on the central limit theorem. The cause of this difficulty is that some irregular phenomena have important influence on the behaviour of the supremum of these random sums. Hence new methods had to be found to tackle these problems. Besides, they wrote an extended version of a former Lecture Note about Wiener-Itô integrals which the Springer Verlag published.

They proved such an inequality for strongly mixing Markov fields which estimates the Wasserstein distance of a measure from the stationary distribution by means of the Wasserstein distances of the conditional measures on cubes of bounded size from the respective local specifications of the stationary distribution. This enables us to find a simple proof for the logarithmic Sobolev inequalities for such Markov fields.

They studied some properties of graphs, important in genetic investigations. They showed that it can be decided in polinomial time whether a degree sequence exists in which the number of edges between a set of points and its complement has a prescribed value. They showed that swaps are not irreducible on the set of realizations, while the swaps and double swaps are irreducible.

They investigated statistical problems related to ergodic sequences. They estimated the residual waiting time for ergodic binary processes. They constructed such a scheme which gives a pointwise consistent estimate for the residual waiting time along a generally sparse sequence of stopping times. However, if the process is a renewal process, the sequence of stopping times considered has density one.

They studied the phase transitions in planar stochastic processes, especially their stationary and near-critical dynamics. They got a better understanding on the near-critical SLE(6) curve. They proved the noise sensitivity of critical bootstrap percolation on the square lattice and noise stability on the random d-regular graph.

They studied some natural problems of financial mathematics. They investigated such problems where the investors follow realistic behavioural patterns (i.e., they are risk averse above a certain desired wealth level and risk-seeking below it). They proved the existence of optimal strategies for such investors in the first non-trivial class of incomplete financial market models in continuous time. For discrete-time investment problems they constructed a counterexample which shows why additional hypotheses in previous works had to be stipulated. They proved another result which characterizes the necessary initial capital to cover a future payment obligation in a market with liquidity constraints (where stocks cannot be bought and sold in arbitrarily large quantities at a given price).

They investigated some random graph models. They extended the class of these models with the “spatial beta” model. In the background of these investigations there is the Barabási model and some other problems related to it.

Applied research

The major part of the research carried out at the Rényi Institute is generated by questions raised by the inner development of mathematics. Nevertheless, together with the exploratory (theoretical) research that the institute conducts, these new results and other fundamental methods of mathematics are applied in other disciplines as well. These include the statistical calculations used in a series of medical papers analyzing birth heart defects and an improved algorithm developed for optimization of cargo shipment timing.

The members of the institute have started collaboration with the Laboratory of Cerebral Cortex Research of the Institute of Experimental Medicine to use network science methods in order to answer the question of what type of connections are responsible for the high frequency, so-called ripple oscillation in the hippocampus. Several students studying at the Budapest Semesters in Mathematics joined the project. The collaboration will continue in 2015, the initial results were presented on a poster at the January 2015 conference of the Hungarian Neuroscience Society.

The Cryptology research group that started in the framework of the first Momentum project and has been given a permanent status had another successful year. The most important event of the year was the Central European Conference on Cryptology organized at the Rényi Institute with active participation of the members of the research group, attracting more than 50 participants, mostly from foreign countries who all found the conference very successful. Part of the presented talks will be published as a research paper in a special issue of Studia Scientiarum Mathematicarum Hungarica. The research group continued the line of research proved to be very successful in the previous years. The main topic is investigation of methods in unconditional cryptography. A fundamental result in graph theory, due to Erdős and Pyber, has been used extensively to obtain lower bounds on the amount of information participants in a secret sharing scheme must receive. They succeeded in giving a significantly simpler proof of the improved version of their result; made far reaching generalizations for hypergraph-based secret sharing schemes, and showed that apart from a factor of two, the method cannot yield better result than they have proved. The shape of the entropy region of several variables has an intimate connection to the best possible bounds on information rate of secret sharing schemes. In this context, an effective algorithm has been developed to enumerate vertices of an implicitly given high dimensional polytope. The algorithm has been applied in financial mathematics to investigate optimal asset distribution in diverse portfolios and has drawn international attention. It was presented as an invited talk at the conference “SET OPTIMIZATION meets FINANCE”. The results about the shape of the entropy region and the methods to achieve them were invited to be presented on a weeklong school to Bogota (Columbia) and to the Stochastic conference in Prague (as an invited talk as well).

Career advancement of researchers

In 2014 one young researcher obtained the PhD degree. At the end of the year 9 members of the Academy (7 according to the statistical number of employees), 33 doctors of MTA (stat. num. 27), 44 researchers with PhD or CSc (stat. numb. 41) worked at the institute, 24 researchers (stat. num. 21) have not yet obtained a degree. Besides the regular employees 12 emeritus research professors (7 academicians, 5 with DSc title) take part in the scientific work of the institute. The institute puts great emphasis on involving young talents – working towards their PhD or just obtaining the degree – into the research work of the institute. In 2014 further 4 young researchers were employed in the new or vacant positions offered by the Academy. Altogether 18 young researchers worked in the institute in 2014. The institute has an agreement with the Central European University. In this framework 12 doctoral students were supervised by members of the institute.

b) Dialog between science and society

Unfortunately, most of the research topics in pure mathematics are not suitable for discussions for the general public. On the other hand, the international success of the researchers has underlined the importance of the research conducted in the institute even in the media.

The researchers of the institute play an important role in popularizing mathematics, giving lectures for high school and university students. The institute regularly organizes an open house during the Festival of Hungarian Science, where high school students and their teachers can get information about the mathematics profession. Members of the institute take part in fostering mathematical talents. In 2014 they have organized several mathematical camps and other events for students interested in the subject. The institute plays a role in giving scientific background for the teachers of specialized mathematics classes in high schools.

III. A presentation of national and international relations

National relations

Researchers of the institute teach part time at many universities both in Budapest and in other cities (Eötvös University, Budapest University of Technology, University of Szeged, Pannon University, etc.). They play an important role in doctoral schools and in Master programs. 16 members of the institute are core members of doctoral schools in various universities, they supervise 49 doctoral students. Especially important is the collaboration between the institute and the Department of Mathematics and its Applications of the Central European University. The lecturers and the supervisors of the Masters and doctoral programs of CEU mainly belong to the institute, including the department chair and the leader of the doctoral program. Also a large part of lecturers of the Budapest Semesters in Mathematics English language study abroad program for North American students belongs to the institute. This program helps to bring the fame of Hungarian mathematics to American universities, and serves as a role model for other international programs. For the institute the close contact with the new generation of mathematicians is of foremost importance. In this spirit 66 members of the institute (69 percent of all researchers) were active in teaching at universities in 2014, that included supervising 2 student research projects, 19 BSc and 19 MSc dissertations.

As part of the renewal program of the Academy, the institute restarted its guest researcher program, which enables university professors and lecturers to spend one or two semesters in the institute freed from their teaching duties. As part of this program four people from the Budapest University of Technology, six from Eötvös University, and one from the University of Szeged joined the research teams of the institute in 2014.

The weekly seminars in the institute are attended regularly by researchers from other institutions, among them some people from universities outside Budapest as well. This way these seminars influence the whole mathematical scene in Hungary.

Members of the Rényi Institute traditionally take part in various Hungarian scientific committees well over proportion. In particular, the Section of Mathematics of the Hungarian Academy of Sciences (MTA) and its committees, the Hungarian Research Fund (OTKA), and the János Bolyai Mathematical Society (BJMT) can be mentioned. The president of the Section of Mathematics of MTA, the chairman and the secretary of the Mathematics Committee, the secretary of the Mathematics Doctoral Committee, one of the vice-chairmen and the secretary of the Bioinformatics Committee, the chairman of the Mathematics and Natural Sciences subcommittee of the Council of the Academy's Research Units, the chairman of the mathematics panel of OTKA, the president of the BJMT, the chairman and the secretary of the Scientific Section of BJMT, the vice-president of the Applied Mathematics Section of BJMT, the secretary general of the Hungarian Society for Bioinformatics are all researchers of the Rényi Institute.

International relations

The researchers of the institute have very extensive international relations. Among the coauthors of the members of the institute one finds mainly foreign mathematicians. Joint projects and jointly organized conferences are also typical.

In 2014 twenty-seven people from the institute were involved in organizing international conferences, some of them even in several occasions. The most important events in the institute were the "Graph limits, groups, and stochastic processes" summer school and workshop, and the "Sum(m)it: 240" conference. The young researchers in the institute have organized the fifth "Emléktábla Workshop" this year. The visits in the framework of the bilateral exchange programs between the Hungarian Academy of Sciences and its partner institutions successfully contributed to the cooperation with foreign partners. With the help of these programs fruitful joint research projects, useful exchange of information, and conference participations were made possible.

Researchers of the institute took part in altogether ten international scientific committees. For example, the vice-president of the European Set Theory Society is a member of the institute. Names of the institute's researchers can be found 156 times on the list of editorial board of various international journals. In 2014 the researchers gave altogether 301 talks at international meetings, many of these were given as an invited or plenary lecture.

In 2014 ten researchers spent more than half a year abroad at the following institutions: University of Chicago (USA), City University of New York (USA), National Science Foundation (USA), University of Washington (USA), École Polytechnique Fédérale de Lausanne (Switzerland), Lancaster University (UK), Georg-August-Universität Göttingen (Germany), Université Catholique de Louvain (Belgium), Université de Toulouse (France).

Financed by the ERC and Momentum grants or from other sources 16 foreign researchers worked in the institute for 1–6 months (altogether that makes up 28 months). They came from France, Italy, Germany, Israel, USA, Canada, and Chile. The number of foreign visitors of the institute – not counting the conference participants, neither the foreign employees – was over 70 in 2014.

IV. Brief summary of national and international research proposals, winning in 2014

National grants

The Rényi Institute, similar to the practices of the previous years, successfully participated – both in terms of the applications and the winning projects in 2014 in the Hungarian OTKA (Hungarian Scientific Research Fund) project proposals. In 2014 further 4 research and 1 postdoc projects won support. The overall OTKA project support of the institute has slightly decreased compared to the previous years.

International grants

The most promising and successful calls for the explanatory (theoretical) research projects of the Rényi Institute are EU European Research Council (ERC) calls and the mobility (Marie Curie) calls of the European Union. 2014 was the transition year from FP7 to H2020, only the results of some FP7 calls were announced and the new calls for H2020 were not yet published. The ERC Consolidator Grant project of the Institute, entitled "Limits of discrete structures", winning in 2013, started in 2014. The research topic of the project is closely related to two, already running Momentum projects and several Marie Curie mobility projects of the institute, and so the resources from these different projects altogether make possible the forming of a significant, international research group.

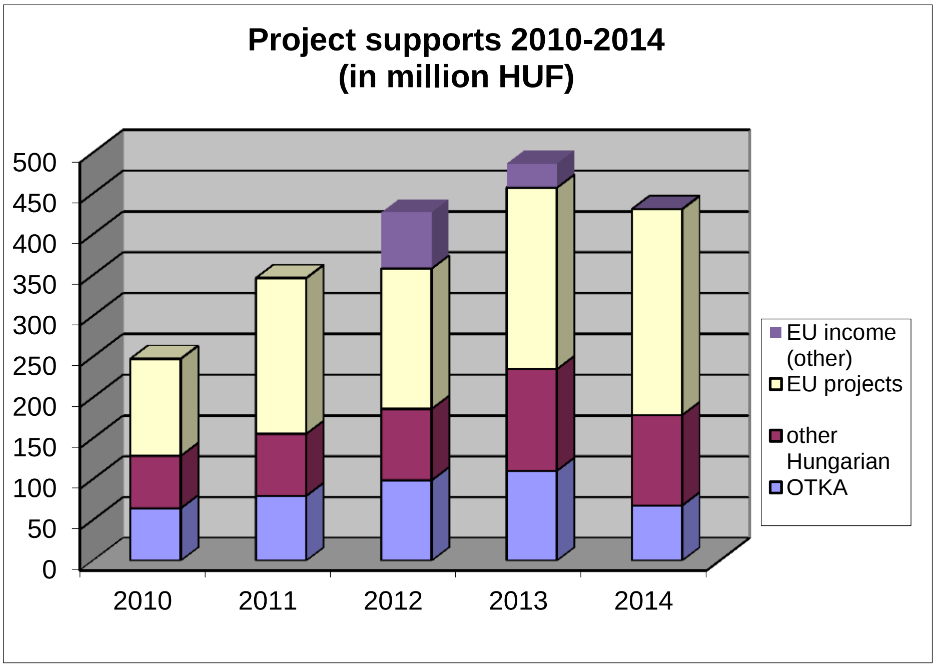
In the framework of the ending FP7 one more mobility project application was submitted, which was funded and already started in 2014 and an earlier winning mobility project was prepared to start early 2015.

For the new H2020 ERC projects one successful Consolidator and an Advanced grant proposal were submitted, with results announced in the first half of 2015.

Altogether, the total research grant income of the institute in 2014 was slightly less than the one in 2013 (but exceeded the grant income of the other previous years). The national (OTKA and Momentum grants) were less than in 2013, partly due to the fact the institute's first Momentum project has become permanent and its support now does not count as a project income. On the other hand, the support of international projects from the EU in 2014 exceeded 25% of the total income of the institute. With the end of the earlier COGANGS project no more other (not direct project funding) income came from the EU. The total project related income in 2014 gave 44% of the total income of the institute.

The running Momentum, OTKA and EU funded projects, together with ERC Consolidator projects started in 2014 and the one awarded early 2015, starting first part of 2015 will ensure that the project related income of the institute will not decrease in 2015.

The following diagram shows the amount of project support received during the last 5 years.



V. List of important publications in 2014

1. Abért M, Glasner Y, Virág B: Kestens theorem for invariant random subgroups. Duke Math J, 163:(3) 465-488 (2014) http://real.mtak.hu/21938
2. Andréka H, Madarász XJ, Németi I, Stannett M, Székely G: Faster than light motion does not imply time travel. Classical Quant Grav, 31:(9) Paper 095005 (2014) http://real.mtak.hu/21939
3. Ánh PN, Gould V, Grillet PA, Márki L: Commutative orders revisited. Semigroup Forum, 89:(2) 336-366 (2014) http://real.mtak.hu/21940
4. Balogh J, Barát J, Gerbner D, Gyárfás A, Sárközy GN: Partitioning 2-edge-colored graphs by monochromatic paths and cycles. Combinatorica, 34:(5) 507-526 (2014) http://real.mtak.hu/21941
5. Bárány I, Pach J: Homogeneous selections from hyperplanes. Journal Comb Theory B, 104: 81-87 (2014) http://real.mtak.hu/21926
6. Bowden J, Crowley D, Stipsicz AI: Contact structures on M x S^2. Math Ann, 358:(1-2) 351-359 (2014) http://real.mtak.hu/9951
7. Csépe Z, Makra L, Voukantsis D, Matyasovszky I, Tusnády G, Karatzas K, Thibaudon M: Predicting daily ragweed pollen concentrations using computational intelligence techniques over two heavily polluted areas in Europe. Sci Total Environ, 476-477: 542-552 (2014) http://real.mtak.hu/21927
8. Domokos M: Invariant theoretic characterization of subdiscriminants of matrices. Linear Multilinear A, 62:(1) 63-73 (2014) http://real.mtak.hu/21928
9. Elekes M, Steprans J: Haar null setsand the consistent reflection of non-meagreness. Can J Math, 66:(2) 303-322 (2014) http://real.mtak.hu/21929

10. Enright J, Stewart L, Tardos G: On list colouring and list homomorphism of permutation and interval graphs. SIAM J Discrete Math, 28:(4) 1675-1685 (2014) http://real.mtak.hu/21930
11. Füredi Z, Jiang T: Hypergraph Turán numbers of linear cycles. Journal Comb Theory A, 123:(1) 252-270 (2014) http://real.mtak.hu/21931
12. Castryck W, Fouvry É, Harcos G, Kowalski E, Michel P, Nelson P, Paldi E, Pintz J, Sutherland AV, Tao T, Xie XF: New equidistribution estimates of Zhang type. Algebr Number Theory, 8: 2067-2199 (2014) http://real.mtak.hu/21932
13. Kroó A, Szabados J: On multivariate incomplete polynomials on starlike domains. Constr Approx, 39:(2) 397-419 (2014) http://real.mtak.hu/21933
14. László T, Némethi A: Ehrhart theory of polytopes and Seiberg-Witten invariants of plumbed 3-manifolds. Geom Topol, 18:(2) 717-778 (2014) http://real.mtak.hu/21934
15. Major P: Multiple Wiener-Itô integrals (revised version): With applications to limit theorems. Springer Verlag, Lect Notes Math, 849: 128 (2014) http://real.mtak.hu/20064
16. Matolcsi M, Ruzsa IZ: Difference sets and positive exponential sums I. General properties. Journal of Fourier Analysis and Applications, 20:(1) 17-41 (2014) http://real.mtak.hu/7885
17. Ozsváth P, Stipsicz AI, Szabó Z: Knots in lattice homology. Commentarii Mathematicii Helvetici, 89:(4) 783-818 (2014) http://real.mtak.hu/21935
18. Petz D, Ruppert L, Szántó A: Conditional SIC-POVMs. IEEE Transactions on Information Theory, 60:(1) 351-356 (2014) http://real.mtak.hu/21936