The asymmetric Beurling–Selberg extremal problem

Abstract

Let $s(x)$ be a real-valued locally integrable function. A classical well-studied problem in Fourier analysis is to find, among all entire functions of exponential type at most 2π, the best approximation F to s, in the sense that

$$\int_{-\infty}^{\infty} |s(x) - F(x)| \mathrm{d}x$$

is minimized. Additionally one may ask that $F(x) \geq s(x)$ for all real x, when the problem is to find the best *majorant* of s, and which is sometimes also referred to as the Beurling–Selberg problem. A notable example is the case $s(x) = \operatorname{sgn} x$, for which the solution, found by Beurling, has numerous applications in Fourier analysis, number theory, and Tauberian theory.

In this talk, we propose the following more general problem, where one instead minimizes

$$(1-\eta)\int_{-\infty}^{\infty} \big(s(x) - F(x)\big)_{+} \mathrm{d}x + \eta \int_{-\infty}^{\infty} \big(s(x) - F(x)\big)_{-} \mathrm{d}x, \quad \text{for some fixed } \eta \in (0,1).$$

(Here $f_{+} = \max\{f, 0\}$, $f_{-} = \max\{-f, 0\}$.) We construct the unique solution for $s(x) = x^{n} \operatorname{sgn} x / n!$ ($n \in \mathbb{N}$), and provide as an application sharp asymmetric finite forms of the Ingham–Karamata Tauberian theorem.

This talk is based on collaborative work with Gregory Debruyne and Jasson Vindas (both from Ghent University).